DATA AND PROBABILITY

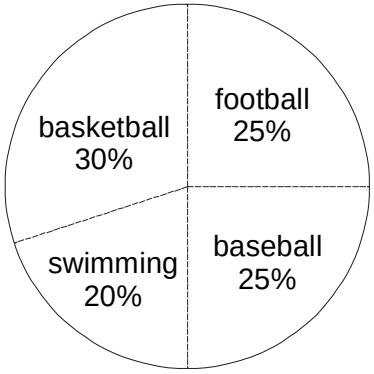
Grade 6

BIG IDEA (1): Formulate questions that can be addressed with data and collect, organize and display data to answer them

CONCEPT	EXPECTATION	EXAMPLE
A Formulate questions	Formulate questions, design studies and collect data about a characteristic	Problem: Students in a class want to determine which type of television shows are the most popular among sixth graders in their school. Which of the following methods should they use to collect their data? A. Survey all the sixth-grade teachers in the school. B. Survey all the sixth-grade students in the school. C. Survey all the students in the school. D. Survey the smallest and largest sixth-grade classes in the school. Answer: B

CONCEPT	EXPECTATION	EXAMPLE
		Problem: For her science fair project, Allison wants to collect data on how the height from which an object is dropped affects the speed at which it falls. Which of the following studies should she do to collect her data? A. Drop different-sized cans from the same height, and record the time it takes them to fall to the ground. B. Drop same-sized cans from different heights, and record the time it takes them to fall to the ground. C. Drop different-sized cans from different heights, and record the time it takes them to fall to the ground. D. Drop the same can from the same height several times, and record the time it takes the can to fall to the ground each time. Answer: B TEACHER NOTES: During the middle-grade years, students should design experiments that will allow them to collect data to answer their questions, learning—in the process—the importance of identifying relevant data, controlling variables, and choosing a sample when it is impossible to collect on every case.[1]

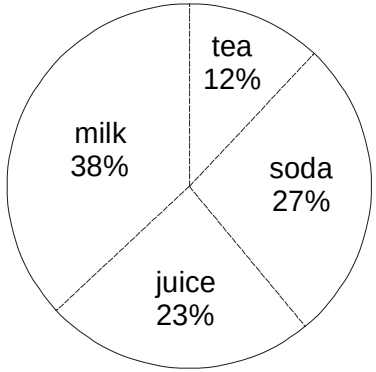
[1] *Navigating through data analysis and probability in grades 6–8* (p. 3). (2003). Reston, VA: National Council of Teachers of Mathematics.

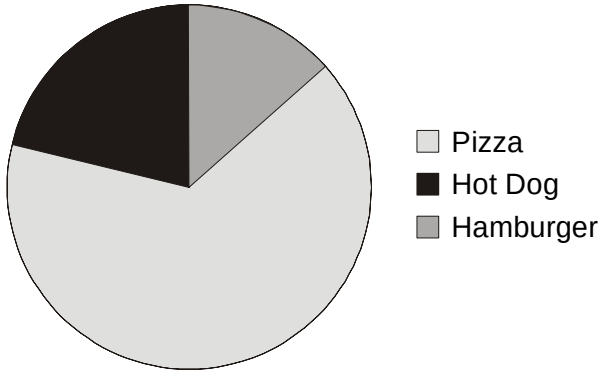
CONCEPT	EXPECTATION	EXAMPLE
C Represent data using physical objects	Interpret circle graphs; create and interpret stem-and-leaf plots	Problem: The circle graph below shows the favorite sport of sixth graders by percentages.  Describe the information presented in this circle graph. Be sure to identify the least and most selected sport. If 100 students were surveyed, how many would say their favorite sport is swimming? Answer: According to the circle graph, the favorite sports of sixth graders are basketball (most favorite), football, baseball, and swimming (least favorite). If 100 students were surveyed, 20 would report swimming as their favorite.

DEFINITION:

stem-and-leaf plot—a method of organizing data from least to greatest using the digits of the greatest place value to group the data. The data is separated into stems (tens) and leaves (ones).[2]

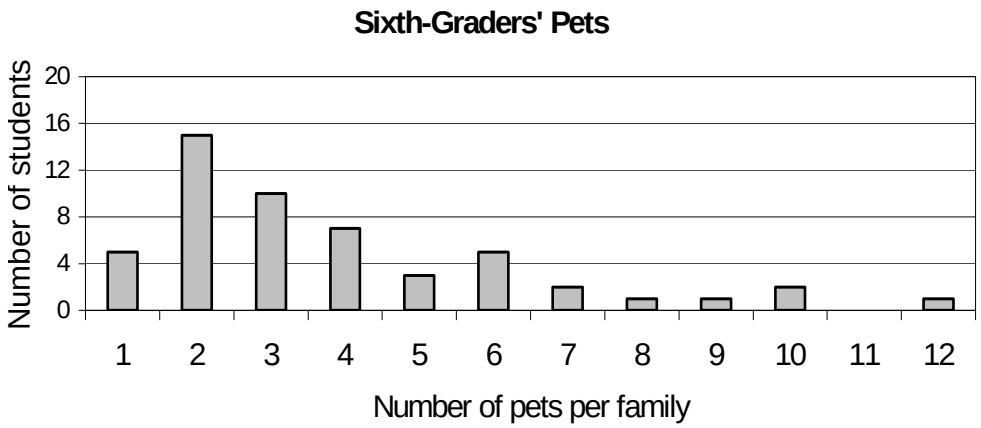
[2] *Math on call: A mathematics handbook* (p. 593). (1998). Wilmington, MA: Great Education Source Group, Inc.

CONCEPT	EXPECTATION	EXAMPLE
		Problem: In a survey of 220 sixth graders, the following items were found to be the students' favorite drinks for lunch. Determine the number of students each of the percentages represents.  Answer: Milk: 220 x 0.38 = 84 Tea: 220 x 0.12 = 26 Soda: 220 x 0.27 = 59 Juice: 220 x 0.23 = 51

CONCEPT	EXPECTATION	EXAMPLE
		Problem:  Based on the circle graph that shows the favorite foods of sixth graders, which of the following statements is true? A. Most sixth graders like hot dogs. B. Most sixth graders like hot dogs more than pizza. C. Most sixth graders like hamburgers more than hot dogs. D. Most sixth grader like pizza. Answer: D

CONCEPT	EXPECTATION	EXAMPLE
		Problem: One student's scores for 12 math assignments are shown below. Create a stem-and-leaf plot to show the scores. 75, 72, 75, 82, 85, 83, 95, 95, 97, 86, 99, 97 Answer: Math Assignment Scores Stem | Leaves 7 | 2, 5, 5 8 | 2, 3, 5, 6 9 | 5, 5, 7, 7, 9 Key: 7|2 represents a score of 72

BIG IDEA (2): Select and use appropriate statistical methods to analyze data

CONCEPT	EXPECTATION	EXAMPLE
A Describe and analyze data	Find the range and measures of center, including median, mode and mean	Problem: 

DEFINITIONS:

mean—the measure of center found by dividing the sum of two or more numbers by the number of addends.[3]

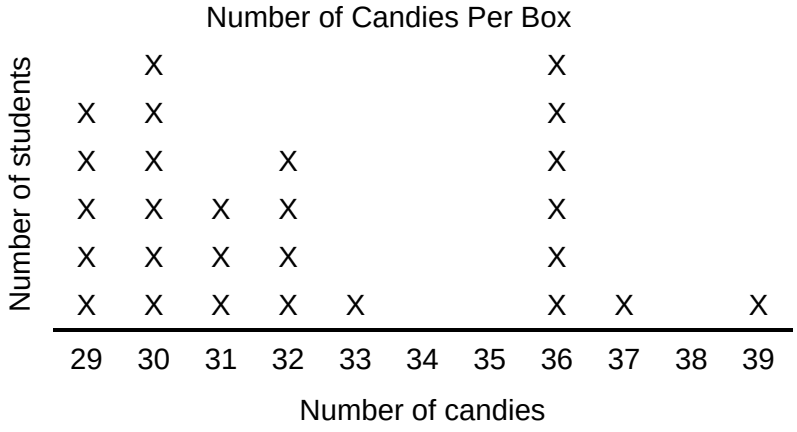
measure of center—measures of center or central tendency describe where data are centered; measures of center include the mean, median, and mode.[4]

median—when the numbers are arranged from least to greatest, the middle number of a set of numbers, or the mean of two middle numbers when the set has two middle numbers. [5]

mode—number that appears most frequently in a set of numbers; there may be one, more than one, or no mode.[6]

range—the difference between the greatest and least value in a set of data.[7]

[3] Cavanagh, M. (2000). *Math to know* (p. 455). Wilmington, MA: Great Source Education Group, Inc.
[4] Billstein, R., Libesking, S., & Lott, J. W. (1998). *A problem solving approach to mathematics for elementary teachers* (p. 492). Reading, MA: Addison-Wesley.
[5] *Math at hand: A mathematics handbook* (p. 527). (1999). Wilmington, MA: Great Source Education Group, Inc.
[6] Cavanagh, M. (2000). *Math to know* (p. 455). Wilmington, MA: Great Source Education Group, Inc.
[7] *Math at hand: A mathematics handbook* (p. 532). (1999). Wilmington, MA: Great Source Education Group, Inc.

CONCEPT	EXPECTATION	EXAMPLE
		According to the graph on the previous page, which of the following statements is true? A. The median number of pets per family is more than the mean. B. The mean number of pets per family is more than the median. C. The mean number of pets per family is two. D. Over half of the families have at least six pets. Answer: B Problem: In one sixth-grade class, each student counted the number of candies in his or her box before eating them. The line plot below shows the number of candies in each box. Each student was asked to write a statement about the data reported on the line plot. Jared wrote the following: The range of the data is 32. Explain why you agree or disagree with Jared's statement. 

CONCEPT	EXPECTATION	EXAMPLE

Answer:
I disagree with Jared's statement. The range of a set of data is the difference between the greatest and least value in the data set. The least number of candies is 29, and the greatest number is 39, so the range would be 39 – 29, or 10.

Problem:
The ages of six of Jenni's friends are12, 12, 13, 11, 12, 12. What is the mean age of her friends? Show your work.

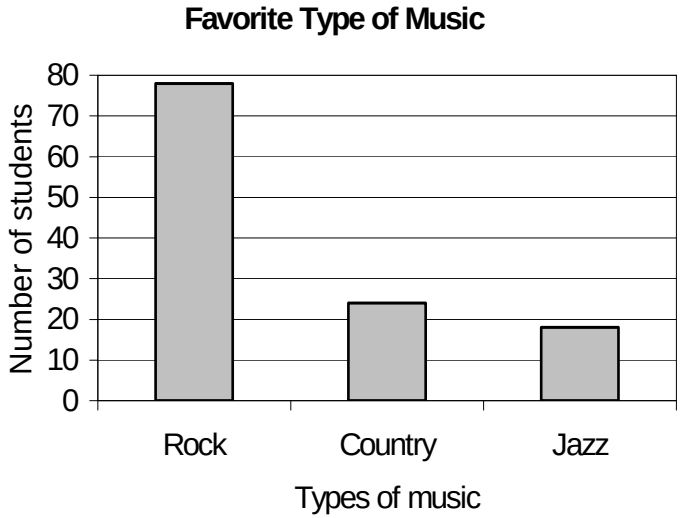
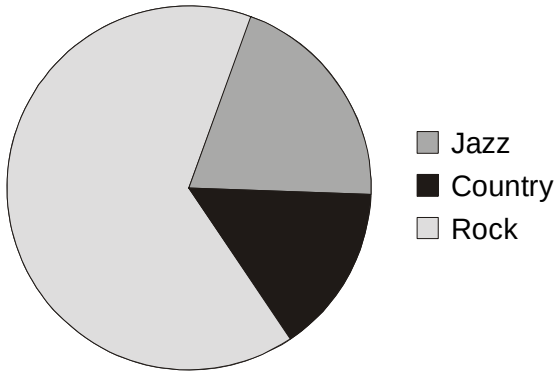
Answer:
12
12 + 12 + 13 + 11 + 12 + 12=72
72/6 = 12

Problem:
The chart below shows the number of hours of TV a sixth grader watched last week. Use the information to find the mean, median, mode, and range of the data in the chart. Show your work.

Day of Week	Number of Hours
Monday	1
Tuesday	1
Wednesday	1
Thursday	2
Friday	5
Saturday	6
Sunday	5

CONCEPT	EXPECTATION	EXAMPLE
		Answer: Mean = 3 hours per week. 1 + 1 + 1 + 2 + 5 + 5 + 6 = 21 21/7 = 3. Range = 5 hours per week. 6 – 1 = 5. Mode = 1, because 1 appears most often. Median = 2 hours per week because the middle number is 2. TEACHER NOTES: In the middle grades, students should learn to use the mean, and continue to use the median and the mode, to describe the center of a set of data. Although the mean often quickly becomes the method of choice for students when summarizing a data set, their knack for computing the mean many not necessarily correspond to a solid understanding of its meaning or purpose—that the mean "evens out" or "balances" a set of data and that the median identifies the "middle" of a data set. In addition, students need to think about the measures of center in relation to the spread of the data distribution. In general, how do changes in data value affect the mean and median of a set of data? These relationships can be effectively demonstrated using software through which students can control a data value and observe how the mean and median are affected. By repeating this process for various data points, they can notice that changing one data value usually does not affect the median at all, unless the moved value is at the middle of the data set or moves across the middle, but that every change in value affects the mean.[8]

[8] National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics* (p. 251). Reston, VA: Author

CONCEPT	EXPECTATION	EXAMPLE
B Compare data representations	Compare different representations of the same data and evaluate how well each representation shows important aspects of the data	Problem: Sixth graders' favorite types of music are shown on the three representations below. Which of these representations would be the best one to use at a PTA meeting for distributing to parents?  

CONCEPT	EXPECTATION	EXAMPLE

Types of Music	Number of Students
Rock	78
Country	24
Jazz	18

Answer:
The bar graph would be the best choice, because it provides a visual and gives the number of students who like each kind of music.

TEACHER NOTES:
"Students should consider how well different graphs represent important characteristics of data sets. For example, they might notice that it is easier to see symmetry or skewness in a graph than in a table of values."[9]

[9] National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics* (p. 251). Reston, VA: Author

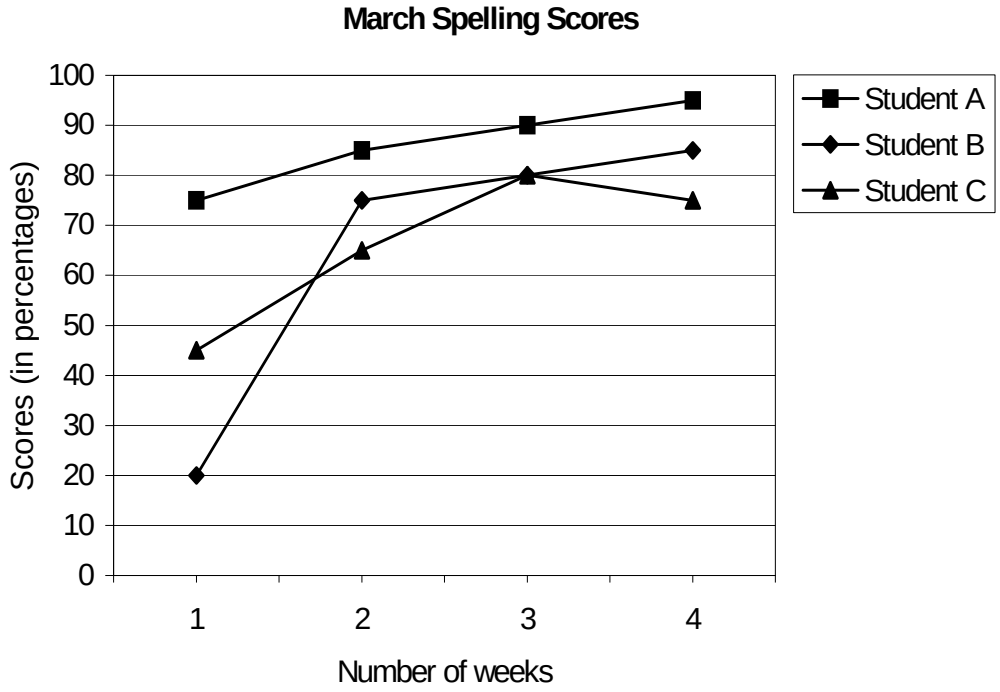
BIG IDEA (3): Develop and evaluate inferences and predictions that are based on data

	CONCEPT	EXPECTATION	EXAMPLE
A	Develop and evaluate inferences	Use observations about differences between 2 samples to make conjectures about the populations from which the samples were taken	Problem: Jan was provided with data on the heights of students in two groups: group A and group B. Do you think that the students in these groups are in the same grade? Why or why not? Group A: 60” 54” 50” 57” 60” 50” 51” 59” 45” Group B: 54” 63” 53” 57” 63” 54” 60” 47” 62” Answer: Group B students are probably in a higher grade, since their mean height is 57 “ while group A has a mean height of 54 “.

DEFINITION:

conjecture—a proposition that is consistent with known data, but has neither been verified nor shown to be false. It is synonymous with hypotheses.[10]

[10] Retrieved February 15, 2005, from mathworld.wolfram.com/Conjecture.htm.

CONCEPT	EXPECTATION	EXAMPLE
		Problem: After looking at the graph of 3 students' scores from a sixth-grade class of 24 students, Gina made a statement that this graph showed that 2 out of 3 sixth graders raised their spelling test scores in March. Explain why you agree or disagree with Gina's statement. 

CONCEPT	EXPECTATION	EXAMPLE
		Answer: Disagree. Gina is basing her statement on only 3 students in a sixth-grade class, and 3 out of 24 is equal to 1 out of 8, not 2 out of 3. In order to check if her statement is true, you would need to look at all 24 sixth graders' spelling scores in March to determine whether two of three (i.e., 16 out of 24 students) raised their spelling scores that month. TEACHER NOTES: "Students should discuss what the relationships they have observed might reveal about the sample, and they should also discuss whether their conjectures about the sample might apply to the larger populations containing the sample. For example, if a sample consists of students from one sixth-grade class in a school, how valid might the inferences made from the sample be for all sixth graders? For all middle-grades students in the school? For all sixth graders in a city? For all sixth graders in the country? Such discussions can suggest further studies students might undertake to test the generality of their conjectures."[11]

[11] National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics* (p. 253). Reston, VA: Author

BIG IDEA (4): Understand and apply basic concepts of probability

	CONCEPT	EXPECTATION	EXAMPLE
A	Apply basic concepts of probability	Use a model (diagrams, list, sample space, or area model) to illustrate the possible outcomes of an event	Problem: Box A contains pieces of paper with the numbers 1, 2, or 3 on them, while box B contains pieces of paper with the numbers 4 or 5 on them. A piece of paper is drawn from each box. Make a sample space to show all possible outcomes. Answer: {1, 4} {1, 5} {2, 4} {2, 5} {3, 4} {3, 5} Problem: At the school carnival, the sixth-grade Math Club has a game titled “Fishing for Common Multiples of 2 and 3.” Players are to draw from fish that are numbered from 1 to 30. Prizes are awarded to those who draw fish that are multiples of both 2 and 3. 1. Make a list of all the possible number outcomes (sample space) of the game. 2. Make a list of all possible winning numbers. 3. Explain whether you think your chance of winning this game is good. Answers: 1. The possible outcomes or sample space includes all the numbers between 1 and 30. 2. The winning numbers are 6, 12, 18, 24, 30. 3. The chances of winning are not very good because you can win on only 5 of the 30 numbers.

CONCEPT	EXPECTATION	EXAMPLE
		TEACHER NOTES: Students should have frequent opportunities to relate their growing understanding of proportionality to simple probabilistic situations from which they can develop notions of chance. As they refine their understanding of chance, or likelihood, that a certain event will occur; they develop a corresponding sense of the likelihood that it will not occur.[12] Outcomes (sample space) include the possible events in a probability simulation.

[12] *Navigating through data analysis and probability in grades 6–8* (pp. ix–x). (2003). Reston, VA: National Council of Teachers of Mathematics.